DAB-003-1012008

DAB-003-1012008 Seat No. ________

B. Sc. (Sem. II) (CBCS) (W.E.F.-2016) Examination

April – 2022

Mathematics : Paper-2(A)

(Geometry, Calculus & Matrix Algebra)

Faculty Code : 003
Subject Code : 1012008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : Answer any five questions.

1 (A) Answer the following questions : **4**

(1) Equation of cylinder with generator parallel to X-axis and enveloping curve is $x^2+y^2+z^2=1$ is ________. Fill in the blank.

(2) Define : Sphere.

(3) Find the equation of the sphere with centre $C(1,2,3)$ and radius $\sqrt{14}$.

(4) Right circular cylinder with radius r intersect XOY plane in a circle then axis of the cylinder is parallel to ________. Fill in the blank.

(B) Obtain the equation of normal to the sphere with (2, –1, 4) and (–2, 2, –2) as extremities of a diameter at the given point (2, –1, 4). **2**

(C) If the plane $-2x+2y-z=k$ touches the sphere $x^2+y^2+z^2+2y-6z+1=0$ then find the value of k. Also find the coordinates of point of contact. **3**

(D) Obtain the equation of cylinder whose generator is parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ passing through the guiding curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, z = 0$. **5**

2 (A) Answer the following questions : **4**

(1) If $x = p\cos\theta, y = p\sin\theta$ then $J\left(\frac{x,y}{p,\theta}\right) \cdot J'\left(\frac{p,\theta}{x,y}\right)$ is equal to ________. Fill in the blank.

(2) If $f_1 = \frac{vw}{u}, f_2 = \frac{wu}{v}, f_3 = \frac{uv}{w}$ then find the value of $\frac{\partial(f_1,f_2,f_3)}{\partial(u,v,w)}$.

(3) Define : Partial derivative of f with respect to x.

(4) State : The Schwartz's Theorem.

(B) Using definition prove that $\lim_{(x,y)\to(2,1)} xy = 2$. **2**

(C) Show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = 8xyzu$ if $u = e^{x^2+y^2+z^2}$. **3**

(D) State and prove Euler's theorem for homogeneous functions of two variables. **5**

3 (A) Answer the following questions : **4**

(1) If error of 3% in E and 2% in R made then the percentage error in $p = \frac{E^2}{R}$ is ________.

(2) Define : Jacobian.

(3) Write conditions for $f(x,y)$ to be maximum.

(4) Define : Saddle point.

(B) If $f(x,y)=x^2y-3y$ then find the approximate value of $f(5.12,3.85)$. **2**

(C) If $u=\dfrac{x+y}{1-xy}, y=\tan^{-1}x+\tan^{-1}y$ then show that $\dfrac{\partial(u,v)}{\partial(x,y)}=0$. **3**

(D) State and prove Taylor's theorem. **5**

4 (A) Answer the following questions : **4**

(1) Define : Matrix

(2) Define : Diagonal Matrix

(3) Define : Null Matrix

(4) Define : Identity Matrix

(B) Define : Minor of a matrix & Adjoint of a square matrix. **2**

(C) Define : Nilpotent Matrix and prove that $A=\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is nilpotent. **3**

(D) Prove that $A=\dfrac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal matrix. **5**

5 (A) Answer the following questions : **4**

(1) Define : Complex Matrix

(2) Define : Trace of Matrix

(3) Define : Unitary Matrix

(4) Define : Hermitian Matrix

(B) Define : Determinant of matrix and Equality of matrix. **2**

(C) Show that the matrix multiplication is associative. **3**

(D) Let $A=[a_{ij}]_{n\times n}, B=[b_{ij}]_{n\times n}$ then show that $(i)\ AdjI=I$, **5**

where I is identity matrix $(ii)\ A(adjA)=|A|I=(adjA).A$.

6 (A) Answer the following questions : **4**

(1) Define : Cylinder

(2) What is the condition that two spheres touch each other externally ?

(3) Definition : Point sphere.

(4) Write the equation of right circular cylinder whose axis is parallel to Y-axis and radius r.

(B) Find the coordinates of the points where the line **2**

$2x-1=y+3=4-z$ intersects the sphere

$x^2+y^2+z^2-6x+8y-4z+4=0$.

(C) Find equation of cylinder where generator is parallel to **3**

$\frac{x}{3}=\frac{y}{1}=\frac{z}{-2}$ through $y^2+2z^2=1; x=3$.

(D) Find the equation of right circular cylinder of which **5**

guiding curve is a circle $x^2+y^2+z^2=4, x+y+z=3$.

7 (A) Answer the following questions : **4**

(1) State Euler's theorem for homogeneous function of two variables.

(2) Define : Maxima and minima.

(3) If u is a homogeneous function of x, y of degree n, then $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=$ ________. Fill in the blank.

(4) If $u=\sin\left(\frac{x^2+y^2}{xy}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$.

(B) If $u=x^4y+y^2z^3$ where $x=rse^t, y=rs^2$ and $z=r^2$ sint then find the value of $\frac{\partial u}{\partial s}$ when $r=2, s=1, t=0$. **2**

(C) $u=e^{xyz}$ then prove that $\frac{\partial^3 u}{\partial x\partial y\partial z}=(1+3xyz+x^2y^2z^2)u$. **3**

(D) If $z=\sin^{-1}\sqrt{\frac{x^3+y^3}{x^2+y^2}}$ then show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{1}{2}\tan z$. **5**

8 (A) Answer the following questions : **4**

(1) Find the maximum value of $x^2+y^2+6x+14$.

(2) $\frac{\partial(u,v)}{\partial(r,s)}\times\frac{\partial(r,s)}{\partial(x,y)}$ is equal to ________. Fill in the blank.

(3) Definition : Extreme value.

(4) Definition : Critical Points.

(B) If $x = r\cos\theta, y = r\sin\theta$ then show that $J = \frac{\partial(x,y)}{\partial(r,\theta)} = r$. **2**

(C) If $W = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ then prove that $x\frac{\partial W}{\partial x} + y\frac{\partial W}{\partial y} + z\frac{\partial W}{\partial z} = 0$. **3**

(D) If $\frac{4}{x} + \frac{9}{y} + \frac{16}{z} = 25$ then using Lagrange's method obtain the value of x, y, z which make x + y + z minimum. **5**

9 (A) Answer the following questions : **4**

(1) Define : Lower triangular matrix with an example.

(2) Define : Characteristic equation of a matrix.

(3) Define : Eigen vector.

(4) Define : Rank of matrix.

(B) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then express $2A^5 - 3A^4 + A^2 - 4I$ as a polynomial of degree one. **2**

(C) If $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then find A^{-1}. **3**

(D) State and prove Cayley-Hamilton theorem. **5**

10 (A) Answer the following questions : **4**

(1) Define : Homogeneous linear equation.

(2) Define : Homogeneous linear equation.

(3) Define : Eigen values.

(4) State Cayley-Hamilton theorem.

(B) Define : Augmented Matrix and Normal form of matrix. **2**

(C) Find the characteristic equation of matrix **3**

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

(D) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then Express polynomial **5**

$A^9 - 2A^8 - 2A^7 - 2A^6 - 2A^5 - 2A^4 - 2A^3 - 2A^2 - 2A + I$
as a quadratic polynomial.